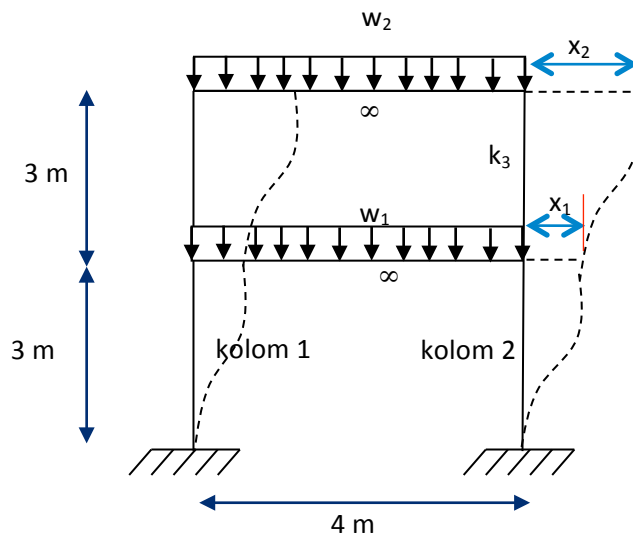


## JAWABAN QUIZ DINAMIKA (4 JUNI 2012)

Perhatikan bangunan bertingkat dua (portal beton) seperti terlihat pada gambar berikut ini.



Diketahui data-data sebagai berikut:

$$w_1 = 500 \text{ kg/m}$$

$$w_2 = 300 \text{ kg/m}$$

$$g = 9,81 \text{ m/s}^2$$

dimensi :

$$\text{kolom 1} = 40/50 \text{ cm}$$

$$\text{kolom 2} = 40/40 \text{ cm}$$

Kekakuan balok dianggap tak terhingga  $\infty$

$$\text{Ebeton} = 2 \times 10^5 \text{ kg/cm}^2$$

### Ditanyakan:

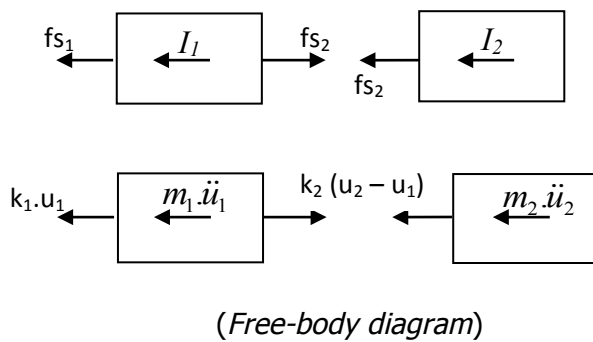
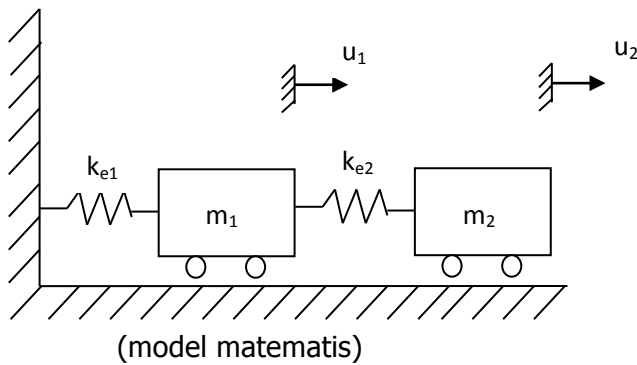
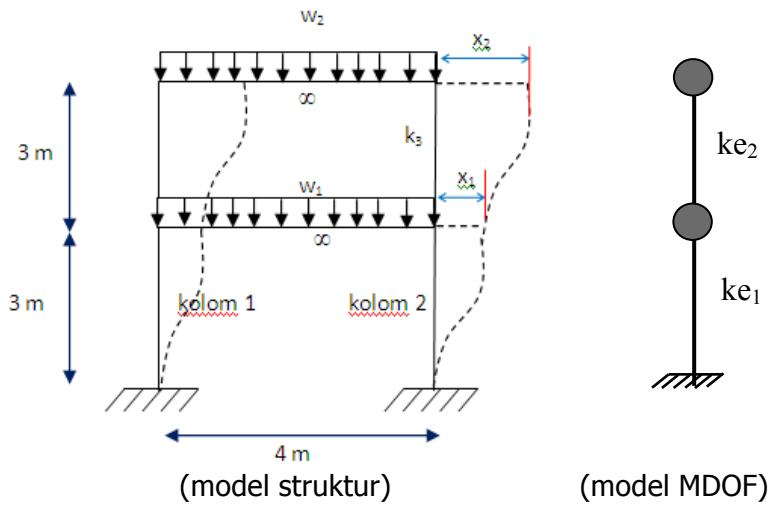
- Gambarkan model DOF, model matematis dan FBD (*Free Body Diagram*)
- Tentukan matriks kekakuan, matriks massa dan persamaan gerak yang terjadi
- Tentukan mode getar dan gambarkan bentuk ragamnya
- Gambarkan respon selama 5 detik untuk ragam 2 dengan syarat awal pada massa 2 sebagai berikut:

$$u(0) = 0 \text{ cm}$$

$$\dot{u}(0) = 0,5 \text{ m/sec}$$

Penyelesaian:

a. Gambarkan model DOF, model matematis dan FBD (*Free Body Diagram*)



b. Penentuan matriks kekakuan, matriks massa dan persamaan gerak yang terjadi

$$[M]\ddot{u} + [K]u = 0$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_{e1} + k_{e2} & -k_{e2} \\ -k_{e2} & k_{e2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = 0$$

$$E = 2.10^5 \text{ kg/cm}^2$$

$$\text{Kekakuan kolom 1} = k_1 = \frac{12EI_c}{h_1^3} = \frac{12(2.10^5) \left( \frac{1}{12} \cdot 40 \cdot 50^3 \right)}{300^3} = 37037,037 \text{ kg/cm}$$

$$\text{Kekakuan kolom 2} = k_2 = \frac{12EI_c}{h_3^3} = \frac{12(2.10^5) \left( \frac{1}{12} \cdot 40 \cdot 40^3 \right)}{300^3} = 18962,963 \text{ kg/cm}$$

$$k_{e1} = k_1 + k_2 \\ = 37037,037 + 18962,963 = 56000 \text{ kg/cm}$$

$$k_{e2} = 37037,037 + 18962,963 = 56000 \text{ kg/cm}$$

$$m_1 = \frac{w_1 \cdot L}{g} = \frac{500 \cdot 4}{9,81} = 203,874 \text{ kg.s}^2/m = 2,04 \text{ kg.s}^2/cm$$

$$m_2 = \frac{w_2 \cdot L}{g} = \frac{300 \cdot 4}{9,81} = 122,324 \text{ kg.s}^2/m = 1,22 \text{ kg.s}^2/cm$$

Matriks massa :

$$[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = \begin{bmatrix} 2,04 & 0 \\ 0 & 1,22 \end{bmatrix} \text{ kg.s}^2/cm$$

Matriks kekakuan :

$$[K] = \begin{bmatrix} k_{e1} + k_{e2} & -k_{e2} \\ -k_{e2} & k_{e2} \end{bmatrix} = \begin{bmatrix} 56000 + 56000 & -56000 \\ -56000 & 56000 \end{bmatrix} = \begin{bmatrix} 112000 & -56000 \\ -56000 & 56000 \end{bmatrix} \text{ kg/cm}$$

c. Tentukan mode getar dan gambarkan bentuk ragamnya

Nilai frekuensi alami ( $\omega_n$ )

$$\det [k - \omega_n^2 m] = 0$$

$$[K] - \omega_n^2 [M] = \begin{bmatrix} 112000 & -56000 \\ -56000 & 56000 \end{bmatrix} - \omega_n^2 \begin{bmatrix} 2,04 & 0 \\ 0 & 1,22 \end{bmatrix} \\ = \begin{bmatrix} 112000 - 2,04\omega_n^2 & -56000 \\ -56000 & 56000 - 1,22\omega_n^2 \end{bmatrix}$$

$$\det([K] - \omega_n^2 [M]) = 0$$

$$\det \begin{bmatrix} 112000 - 2,04\omega_n^2 & -56000 \\ -56000 & 56000 - 1,22\omega_n^2 \end{bmatrix} = 0$$

$$(112000 - 2,04\omega_n^2)(56000 - 1,22\omega_n^2) - (-56000)(-56000) = 0$$

$$6,272 \cdot 10^9 - 136640\omega_n^2 - 114240\omega_n^2 + 2,489\omega_n^4 - 3,136 \cdot 10^9 = 0$$

$$2,489\omega_n^4 - 250880\omega_n^2 + 3,136 \cdot 10^9 = 0$$

Persamaan penyelesaian:

$$a = 2,489$$

$$b = -250880$$

$$c = 3,136 \cdot 10^9$$

$$\omega_n^2 = \frac{-(-250880) \pm \sqrt{(-250880)^2 - 4(2,489)(3,136 \cdot 10^9)}}{2(2,489)}$$

$$= \frac{250880 \pm 178097,6092}{4,978}$$

$$\omega_1^2 = \frac{250880 - 178097,6092}{4,978} = 14620,81 \Rightarrow \omega_1 = 120,917 \text{ rad/s}$$

$$\omega_2^2 = \frac{250880 + 178097,6092}{4,978} = 86174,69 \Rightarrow \omega_2 = 293,555 \text{ rad/s}$$

Pola natural untuk sistem I diperoleh dengan mensubstitusikan  $\square_n = \square_1$

$$[k - \omega_n^2 m] \{\phi_n\} = 0 \rightarrow \omega_1^2 = 14620,81$$

$$\begin{bmatrix} 112000 - 2,04\omega_n^2 & -56000 \\ -56000 & 56000 - 1,22\omega_n^2 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0 \rightarrow \omega_1^2 = 14620,81$$

$$\begin{bmatrix} 112000 - 2,04\omega_1^2 & -56000 \\ -56000 & 56000 - 1,22\omega_1^2 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

$$\begin{bmatrix} 112000 - 2,04(14620,81) & -56000 \\ -56000 & 56000 - 1,22(14620,81) \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

$$\begin{bmatrix} 82173,548 & -56000 \\ -56000 & 38162,612 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

Pola natural ditentukan dengan menentukan satu satuan harga untuk salah satu pola.

Misalkan  $\phi_{21} = 1$ , sehingga diperoleh nilai  $\phi_{11}$  dari analisis berikut ini

$$\begin{bmatrix} 82173,548 & -56000 \\ -56000 & 38162,612 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ 1 \end{Bmatrix} = 0 = 0$$

$$82173,548\phi_{11} - 56000 = 0$$

$$-56000\phi_{11} + 38162,612 = 0 \quad +$$

$$26173,548\phi_{11} - 17837,388 = 0$$

$$\phi_{11} = 0,6815$$

Pola natural untuk sistem II diperoleh dengan mensubstitusikan  $\square_n = \square_2$

$$[k - \omega_n^2 m] \{\phi_n\} = 0 \rightarrow \omega_2^2 = 86174,69$$

$$\begin{bmatrix} 112000 - 2,04\omega_n^2 & -56000 \\ -56000 & 56000 - 1,22\omega_n^2 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0 \rightarrow \omega_2^2 = 86174,69$$

$$\begin{bmatrix} 112000 - 2,04\omega_2^2 & -56000 \\ -56000 & 56000 - 1,22\omega_2^2 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

$$\begin{bmatrix} 112000 - 2,04(86174,69) & -56000 \\ -56000 & 56000 - 1,22(86174,69) \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

$$\begin{bmatrix} -63796,368 & -56000 \\ -56000 & -49133,122 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

Pola natural ditentukan dengan menentukan satu satuan harga untuk salah satu pola.

Misalkan  $\phi_{22} = 1$ , sehingga diperoleh nilai  $\phi_{12}$  dari analisis berikut ini

$$\begin{bmatrix} -63796,368 & -56000 \\ -56000 & -49133,122 \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ 1 \end{Bmatrix} = 0$$

$$-63796,368\phi_{12} - 56000 = 0$$

$$-56000\phi_{12} - 49133,122 = 0 +$$

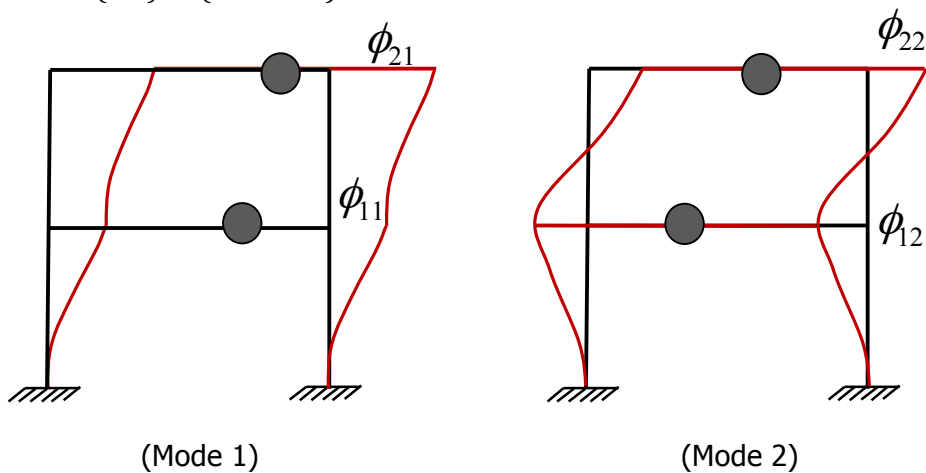
$$-119796,368\phi_{12} - 105133,122 = 0$$

$$\phi_{12} = -0,8776$$

Sehingga, pola natural dari sistem tersebut adalah:

$$\{\phi_1\} = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = \begin{Bmatrix} 0,6815 \\ 1 \end{Bmatrix}$$

$$\{\phi_2\} = \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = \begin{Bmatrix} -0,8776 \\ 1 \end{Bmatrix}$$



Kontrol Kondisi Orthogonalitas:

$$\begin{aligned} \{\phi_1\}^T [K] \{\phi_2\} &= 0 \\ \{0,6815 \quad 1\} \begin{bmatrix} 112000 & -56000 \\ -56000 & 56000 \end{bmatrix} \begin{bmatrix} -0,8776 \\ 1 \end{bmatrix} &\approx 0 \quad OK \\ \{\phi_1\}^T [M] \{\phi_2\} &= 0 \\ \{0,6815 \quad 1\} \begin{bmatrix} 2,04 & 0 \\ 0 & 1,22 \end{bmatrix} \begin{bmatrix} -0,8776 \\ 1 \end{bmatrix} &\approx 0 \quad OK \end{aligned}$$

d. Gambarkan respon selama 5 detik untuk ragam 2 dengan syarat awal sebagai berikut:

$$u(0) = 0 \text{ cm}$$

$$\dot{u}(0) = 0,5 \text{ m/sec} = 50 \text{ cm/sec}$$

Sehingga:

$$\{\phi_2\} = \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = \begin{Bmatrix} -0,8776 \\ 1 \end{Bmatrix}$$

$$\omega_2 = 293,555 \text{ rad/s}$$

Persamaan gerak:

$$u(0) = \sum_{n=1}^n \phi_n \cdot q_n(0)$$

$$\dot{u}(0) = \sum_{n=1}^n \phi_n \cdot \dot{q}_n(0)$$

$$\mathbf{u}(\mathbf{t}) = \sum_{n=1}^n \phi_n \left[ q_n(0) \cos \omega_n t + \frac{\dot{q}_n(0)}{\omega_n} \sin \omega_n t \right] = \sum_{n=1}^n \phi_n \cdot q_n(t)$$

$$\mathbf{u}(\mathbf{t}) = \left[ u(0) \cos \omega_2 t + \frac{\dot{u}(0)}{\omega_2} \sin \omega_2 t \right]$$

$$\begin{aligned} \{u(t)\} &= \left[ 0 \cdot \cos(293,555t) + \frac{50}{293,555} \sin(293,555t) \right] \\ &= [0,1703 \cdot \sin(293,555t)] \end{aligned}$$

Tabel perhitungan respon struktur

No	Time t (s)	Sin $\omega_c t$	u(t)
1	0	0.000	0.000
2	0.5	0.769	0.131
3	1	-0.983	-0.167
4	1.5	0.488	0.083
5	2	0.360	0.061
6	2.5	-0.947	-0.161
7	3	0.852	0.145
8	3.5	-0.141	-0.024
9	4	-0.671	-0.114
10	4.5	0.999	0.170
11	5	-0.606	-0.103

Hubungan antara u (t) dan waktu (t)

