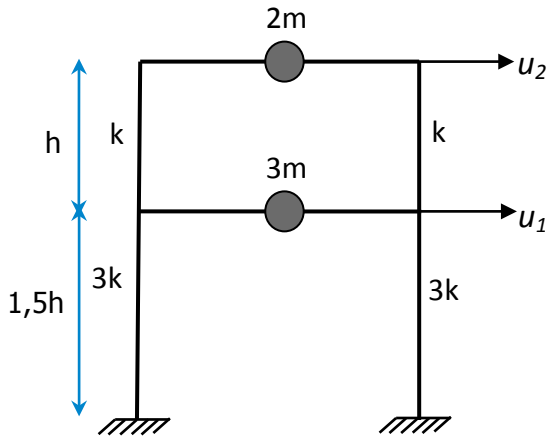


JAWABAN TUGAS XII DINAMIKA

Tentukan frekuensi alami dan pola natural (natural mode) pada sistem struktur di bawah ini.



Penyelesaian:

➤ *Matriks massa*

$$\mathbf{m} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = \begin{bmatrix} 3m & 0 \\ 0 & 2m \end{bmatrix} = m \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

➤ *Matriks kekakuan*

$$k_1 = \frac{12(3EI_c)}{(1,5h)^3} + \frac{12(3EI_c)}{(1,5h)^3} = \frac{6(12EI_c)}{3,375h^3} = 1,78 \left(\frac{12EI_c}{h^3} \right) = 1,78k$$

$$k_2 = \frac{12(EI_c)}{(h)^3} + \frac{12(EI_c)}{(h)^3} = 2 \left(\frac{12EI_c}{h^3} \right) = 2k$$

$$k = \frac{12EI_c}{h^3}$$

$$\mathbf{k} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} = \begin{bmatrix} 1,78k + 2k & -2k \\ -2k & 2k \end{bmatrix} = k \begin{bmatrix} 3,78 & -2 \\ -2 & 2 \end{bmatrix}$$

➤ *Nilai frekuensi alami (ω_n)*

$$\det [\mathbf{k} - \mathbf{m} \omega_n^2] = 0$$

$$\begin{aligned} [\mathbf{K}] - [\mathbf{M}] \omega_n^2 &= \begin{bmatrix} 3,78k & -2k \\ -2k & 2k \end{bmatrix} - \begin{bmatrix} 3m & 0 \\ 0 & 2m \end{bmatrix} \omega_n^2 \\ &= \begin{bmatrix} 3,78k - 3m\omega_n^2 & -2k \\ -2k & 2k - 2m\omega_n^2 \end{bmatrix} \end{aligned}$$

$$\det([K] - [M]\omega_n^2) = 0$$

$$\det \begin{bmatrix} 3,78k - 3m\omega_n^2 & -2k \\ -2k & 2k - 2m\omega_n^2 \end{bmatrix} = 0$$

$$(3,78k - 3m\omega_n^2)(2k - 2m\omega_n^2) - (-2k)(-2k) = 0$$

$$7,56k^2 - 7,56km\omega_n^2 - 6km\omega_n^2 + 6m^2\omega_n^4 - 4k^2 = 0$$

$$6m^2\omega_n^4 - 13,56km\omega_n^2 + 3,56k^2 = 0$$

Persamaan penyelesaian:

$$a = 6m^2$$

$$b = -13,56km$$

$$c = 3,56k^2$$

$$\omega_n^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-13,56km) \pm \sqrt{(-13,56km)^2 - 4(6m^2)(3,56k^2)}}{2(6m^2)}$$

$$= \frac{13,56km \pm \sqrt{183,8736k^2m^2 - 85,44k^2m^2}}{12m^2}$$

$$= \frac{13,56km \pm 9,92km}{12m^2}$$

$$\omega_1^2 = \frac{13,56km - 9,92km}{12m^2} = \frac{0,303k}{m}$$

$$\omega_1 = \sqrt{\frac{0,303k}{m}}$$

$$\omega_2^2 = \frac{13,56km + 9,92km}{12m^2} = \frac{1,957k}{m}$$

$$\omega_2 = \sqrt{\frac{1,957k}{m}}$$

➤ Pola natural untuk sistem I diperoleh dengan mensubstitusikan $\square_n = \square_1$

$$\begin{bmatrix} 3,78k - 3m\omega_n^2 & -2k \\ -2k & 2k - 2m\omega_n^2 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0 \rightarrow \omega_1^2 = \frac{0,303k}{m}$$

$$\begin{bmatrix} 3,78k - 3m\left(\frac{0,303k}{m}\right) & -2k \\ -2k & 2k - 2m\left(\frac{0,303k}{m}\right) \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

$$\begin{bmatrix} 2,871k & -2k \\ -2k & 1,394k \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

Pola natural ditentukan dengan menentukan satu satuan harga untuk salah satu pola.

Misalkan $\phi_{21} = 1$, sehingga diperoleh nilai ϕ_{11} dari analisis berikut ini

$$\begin{bmatrix} 2,871k & -2k \\ -2k & -1,394k \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = 0$$

$$\begin{bmatrix} -2,871k & -2k \\ -2k & 1,394k \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ 1 \end{Bmatrix} = 0$$

$$2,871k\phi_{11} - 2k = 0$$

$$\frac{-2k\phi_{11} + 1,394k = 0}{0,871k\phi_{11} - 0,606k = 0}$$

$$0,871k\phi_{11} - 0,606k = 0$$

$$0,871k\phi_{11} = 0,606k$$

$$\phi_{11} = 0,696$$

➤ Pola natural untuk sistem II diperoleh dengan mensubstitusikan $\square_n = \square_2$

$$\begin{bmatrix} 3,78k - 3m\omega_n^2 & -2k \\ -2k & 2k - 2m\omega_n^2 \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = 0 \rightarrow \omega_2^2 = \frac{1,957k}{m}$$

$$\begin{bmatrix} 3,78k - 3m\left(\frac{1,957k}{m}\right) & -2k \\ -2k & 2k - 2m\left(\frac{1,957k}{m}\right) \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = 0$$

$$\begin{bmatrix} -2,091k & -2k \\ -2k & -1,914k \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = 0$$

Pola natural ditentukan dengan menentukan satu satuan harga untuk salah satu pola.

Misalkan $\phi_{22} = 1$, sehingga diperoleh nilai ϕ_{12} dari analisis berikut ini

$$\begin{bmatrix} -2,091k & -2k \\ -2k & -1,914k \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ 1 \end{Bmatrix} = 0$$

$$-2,091k\phi_{12} - 2k = 0$$

$$\frac{-2k\phi_{12} - 1,914k = 0}{-0,091k\phi_{12} - 0,086k = 0}$$

$$-0,091k\phi_{12} - 0,086k = 0$$

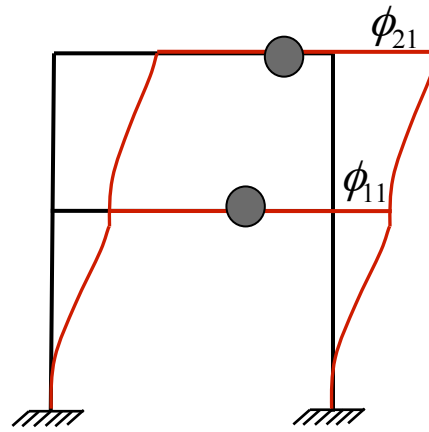
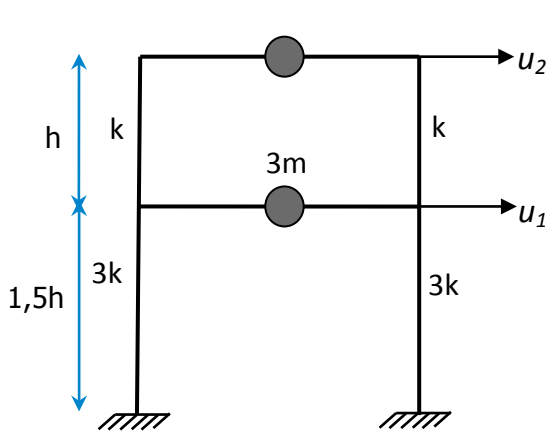
$$-0,091k\phi_{12} = 0,086k$$

$$\phi_{12} = -0,945$$

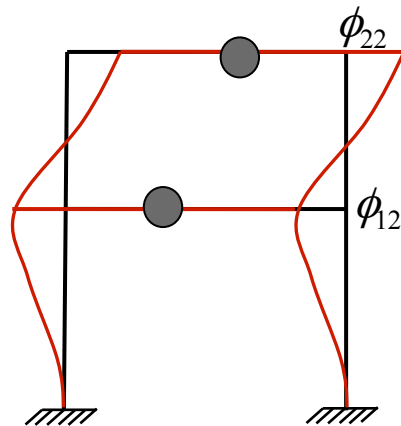
Sehingga, pola natural dari sistem tersebut adalah:

$$\begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = \begin{Bmatrix} 0,696 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = \begin{Bmatrix} -0,945 \\ 1 \end{Bmatrix}$$



Pola Natural I



Pola Natural II

➤ *Kontrol kondisi orthogonal*

$$\phi_1^T \kappa \phi_2 = 0$$

$$k \begin{Bmatrix} \phi_{11} & \phi_{21} \end{Bmatrix} \begin{bmatrix} 3,78 & -2 \\ -2 & 2 \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = 0$$

$$k \begin{Bmatrix} 0,696 & 1 \end{Bmatrix} \begin{bmatrix} 3,78 & -2 \\ -2 & 2 \end{bmatrix} \begin{Bmatrix} -0,945 \\ 1 \end{Bmatrix} = 0$$

$$k \begin{Bmatrix} 0,696 & 1 \end{Bmatrix} \begin{Bmatrix} -5,572 \\ 3,89 \end{Bmatrix} = 0,01k \approx 0$$

$$\phi_1^T m \phi_2 = 0$$

$$m \begin{Bmatrix} \phi_{11} & \phi_{21} \end{Bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = 0$$

$$m \begin{Bmatrix} 0,696 & 1 \end{Bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} -0,945 \\ 1 \end{Bmatrix} = 0$$

$$m \begin{Bmatrix} 0,696 & 1 \end{Bmatrix} \begin{Bmatrix} -2,835 \\ 2 \end{Bmatrix} = 0,02m \approx 0$$

